Towards Compositional and Generative Tensor Optimizations

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Tensors in Computational Fluid Dynamics (CFD)

- Loop characteristics:
  - 3 to 4 dimensions nesting
  - Few iterations per dimension (e.g., 17 or 33 iterations)
- Type of computations:
  - Tensor contractions
  - Outer products
  - Element-wise multiplications
- Computations on each element of a structured mesh

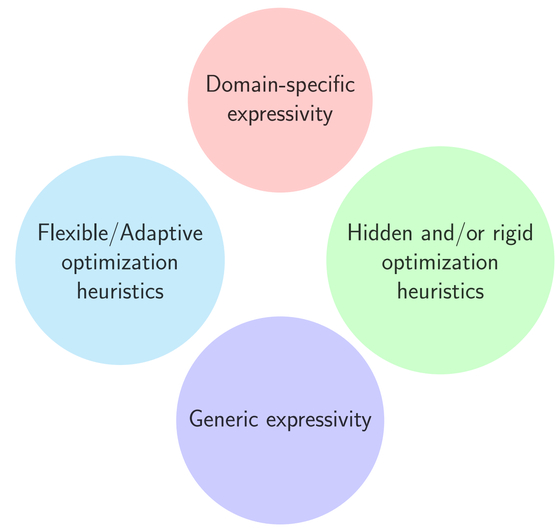
Inverse Helmholtz

$$t_{ijk} = \sum_{l,m,n} A^T_{kn} \cdot A^T_{jm} \cdot A^T_{il} \cdot u_{lmn}$$

$$p_{ijk} = D_{ijk} \cdot t_{ijk}$$

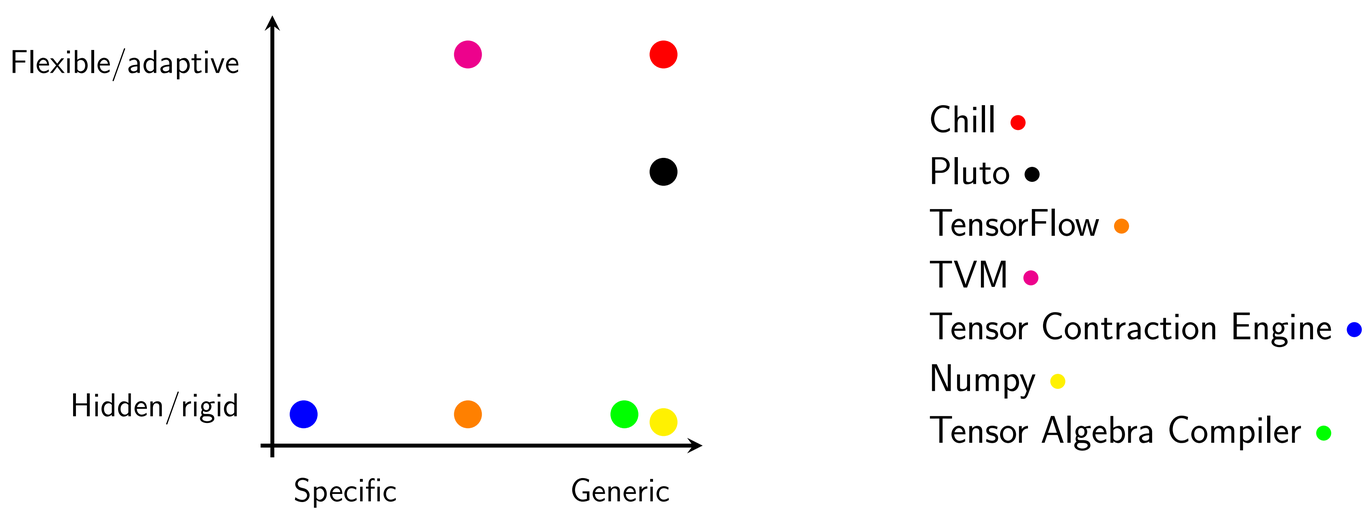
$$v_{ijk} = \sum_{l,m,n} A_{kn} \cdot A_{jm} \cdot A_{il} \cdot p_{lmn}$$

Tensor Optimization Frameworks



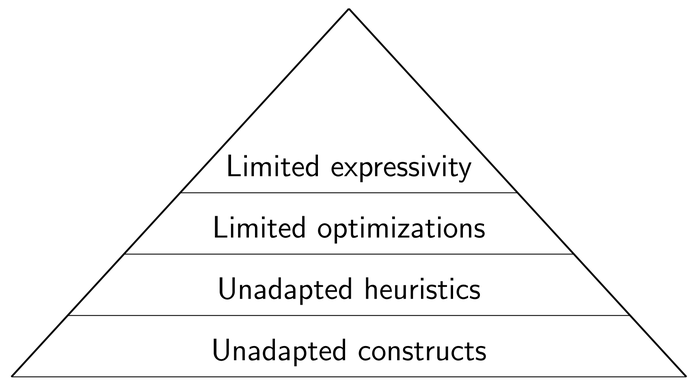
Related Work

- Different levels of expressiveness and control on optimizations



Optimizing CFD Kernels with Existing Tools

- Several limitations
- Few opportunities for adaptations



Should we create yet another domain-specific solution?

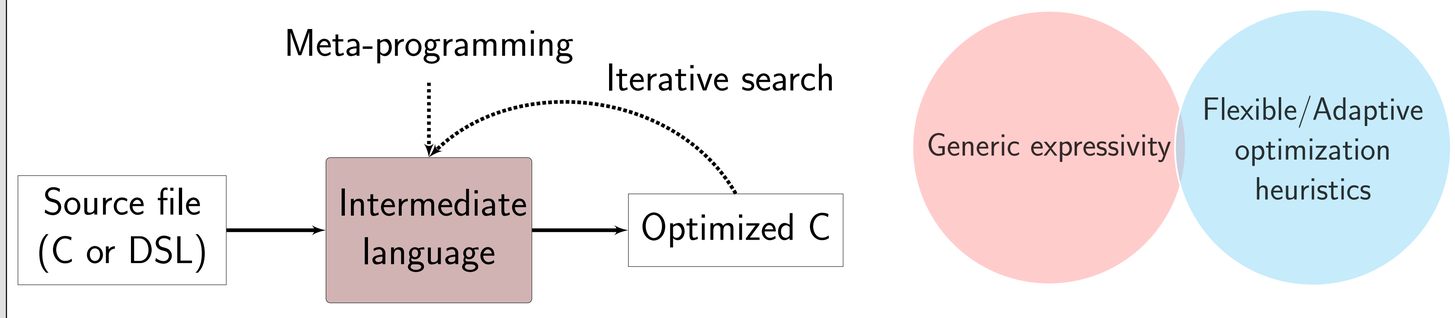
Goal

A cross-domain intermediate language for tensor optimizations

Intermediate Language

- Modular constructs
- First-class citizens:
  - Arrays
  - Tensor operators
  - Loop iterators
  - Transformations

Envisioned Tool



Search Space Exploration

- Evaluation order of tensor contractions
- Fusions
- Permutations
- Vectorization
- Collapsing
- Unrolling

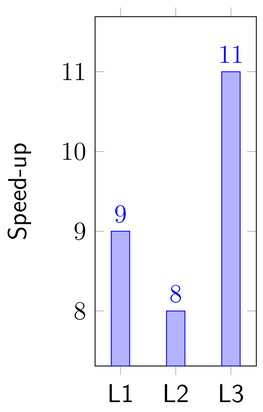
Inverse Helmholtz by Example

```
# Basic array declaration
A = array(2, double, [N, N])
u = array(3, double, [N, N, N])
D = array(3, double, [N, N, N])

# Transposition
At = vtranspose(A, 1, 2)

# Tensor contractions
tmp1 = contract(At, u, [2, 1])
tmp2 = contract(At, tmp1, [2, 2])
tmp3 = contract(At, tmp2, [2, 3])

# Element-wise multiplication
tmp4 = entrywise(D, tmp3)
```

```
# Tensor contractions
tmp5 = contract(A, tmp4, [2, 1])
tmp6 = contract(A, tmp5, [2, 2])
v = contract(A, tmp6, [2, 3])

# Iterator declaration
i1 = iterator(0, N, 1)
i2 = iterator(0, N, 1)
# ... other iterator declarations

# Association of iterators
# to computations
build(D, [td1, td2, td3])
build(tmp1, [i1, i2, i3, i4])
# Also applies to tmp2, ..., tmp6
build(v, [k12, k22, k32, k42])
```

```
# Loop interchanges
interchange(i4, i3)
interchange(i4, i2)
interchange(j2, j1)
interchange(j1, j4)

# Transpositions
tmp2t = vtranspose(tmp2, 1, 2)
replace_array(j3, tmp2, tmp2t)
replace_array(k4, tmp2, tmp2t)
tmp3t = vtranspose(tmp3, 1, 3)
replace_array(k4, tmp3, tmp3t)
interchange(k3, k2)
# ... other optimizations
```

Example of assessment: Different heuristics of loop interchanges (+ parallelization)



- Variant L1: Loop interchanges only;
- Variant L2: Loop interchanges + data transpositions with copying;
- Variant L3: Loop interchanges + data transpositions without copying.

Baseline: sequential execution (3.32s). Machine: 24-core Intel(R) Xeon(R) CPU E5-2680 v3 @ 2.50GHz (Haswell)

Future Work

- Applications to other domains
- Syntax refinement
- Formal semantics

This work was partially funded by the German Research Council (DFG) through the Cluster of Excellence 'Center for Advancing Electronics Dresden' (cfaed) and by PSL Research University through the ACOPAL project.