Meta-programming for Cross-Domain Tensor Optimizations

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Design and semantics of a tensor optimization meta-language

- Functional transformation meta-language with high-level abstractions for tensor computations to enable the composition of different type of transformations (e.g. loop, algebraic or layout transformations).
- Formal specification of tensor operations and loop transformations using denotational semantics.
- Examples of use: empirical tuning engines, meta-programming optimizations by experts.

TeML Grammar

⟨*program*⟩ ::= ⟨*stmt*⟩ ⟨*program*⟩
| ϵ

⟨*stmt*⟩ ::= ⟨*id*⟩ = ⟨*expression*⟩
| ⟨*id*⟩ = @⟨*id*⟩ : ⟨*expression*⟩
| `codegen` (⟨*ids*⟩)
| `init` (...)

⟨*expression*⟩ ::= ⟨*Texpression*⟩
| ⟨*Lexpression*⟩

⟨*Texpression*⟩ ::= `tensor` ([⟨*ints*⟩])
| `eq` (⟨*id*⟩, ⟨*iters*⟩? → ⟨*iters*⟩)
| `vop` (⟨*id*⟩, ⟨*id*⟩, [⟨*iters*⟩?, ⟨*iters*⟩?])
| `op` (⟨*id*⟩, ⟨*id*⟩, [⟨*iters*⟩?, ⟨*iters*⟩?] → ⟨*iters*⟩)

⟨*Lexpression*⟩ ::= `build` (⟨*id*⟩)
| `stripmine` (⟨*id*⟩, ⟨*int*⟩, ⟨*int*⟩)
| `interchange` (⟨*id*⟩, ⟨*int*⟩, ⟨*int*⟩)
| ...

⟨*iters*⟩ ::= [⟨*ids*⟩]

⟨*ids*⟩ ::= ⟨*id*⟩ (, ⟨*id*⟩)*

⟨*ints*⟩ ::= ⟨*int*⟩ (, ⟨*int*⟩)*

Code Sample

```
# -- Begin program specification
w = tensor(double, [13])
u = tensor(double, [13, 13, 13])
L = tensor(double, [13, 13])
M_ = outerproduct([w, w, w])
Lh = div(L, w, [[i1, i2], [i2]] -> [i1, i2])
M  = entrywise_mul(M_, u)
r1 = contract(Lh, M, [[2, 1]])
r2 = contract(Lh, M, [[2, 2]])
r3 = contract(Lh, M, [[2, 3]])
# -- End program specification
# Code generation without transformations
l1 = build(M_)
l2 = build(Lh)
l3 = build(M)
l4 = build(r1)
l5 = build(r2)
l6 = build(r3)
codegen([l1, l2, l3, l4, l5, l6])
# Code generation with loop fusions only
l7 = fuse(l4, l5, 3)
l8 = fuse(l7, l6, 3)
codegen([l1, l2, l3, l7])
# Code generation with fusion, parallelism and vectorization
l9 = parallelize(l1, 1, None)
l10 = parallelize(l2, 1, None)
l11 = parallelize(l3, 1, None)
l12 = parallelize(l8, 1, None)
l13 = vectorize(l9, 3)
l14 = vectorize(l10, 2)
l15 = vectorize(l11, 3)
codegen([l13, l14, l15, l12])
```

Semantics Foundations

Domains and state

$$\mathbf{T} = \{\, \langle (op, S, I), ts\rangle \mid (ts = []) \vee (ts = [t_1, \ldots, t_k] \wedge t_i \in \mathbf{T}) \}$$
$$\mathbf{L} = \{\, \langle id, [x_1, \ldots, x_k]\rangle \mid x_i \in \mathbf{L} \cup \mathbf{T} \,\}$$
$$\sigma : identifier \rightarrow (\mathbf{T} + \mathbf{L})$$

Valuation functions

$$\mathcal{P}_{prog}[\![s\ p]\!] = \mathcal{P}_{prog}[\![p]\!] \circ \mathcal{P}_{stmt}[\![s]\!]$$

$$\mathcal{E}_t[\![\texttt{tensor}(S)]\!] = \lambda\sigma.\langle(\square, S, \epsilon), []\rangle$$

$$\mathcal{E}_t[\![\texttt{eq}(t, I_0 \rightarrow I_1)]\!] =$$
$$\lambda\sigma.\text{let } \langle(op, S, I'), ys\rangle = \sigma(t)$$
$$y = \langle(op, S, I''), ys\rangle$$
$$x = \langle(\square, S', I_1), []\rangle$$
$$\text{in } \langle(=, \bullet, \bullet), [x, y]\rangle\,,$$
$$\text{where } \begin{cases} I' \neq \epsilon \wedge I'' = I'\,, \text{ if } I_0 = \epsilon \\ I' = \epsilon \wedge I'' = I_0\,, \text{ if } I_0 \neq \epsilon \end{cases}$$

$$\mathcal{E}_l[\![\texttt{build}(t)]\!] = \lambda\sigma.\text{let } r = \text{“number of iterators in } \sigma(t)\text{”}$$
$$i_k = (0, ub_k, 1) \text{ for } k = 1, \ldots, r$$
$$\text{in } \langle i_1, \ldots, \langle i_r, [\sigma(t)]\rangle \ldots\rangle\,,$$
$$\text{where } \sigma(t) = \langle(=, \bullet, \bullet), [x, y]\rangle$$

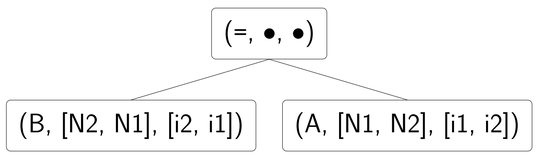
$$\mathcal{E}_l[\![\texttt{stripmine}(l, r, v)]\!] =$$
$$\lambda\sigma.\text{let } \langle i_1, \ldots \langle i_r, xs\rangle \ldots\rangle = \sigma(l)$$
$$(b, e, 1) = i_r$$
$$i'_r = (0, (e - b)/v - 1, 1)$$
$$i'_{r+1} = (b + v \cdot i'_r, b + v \cdot i'_r + (v - 1), 1)$$
$$\text{in } \langle i_1, \ldots \langle i'_r, [\langle i'_{r+1}, xs\rangle]\rangle \ldots\rangle$$

$$\mathcal{E}_l[\![\texttt{interchange}(l, r_1, r_2)]\!] =$$
$$\lambda\sigma.\text{let } \langle i_1, \ldots \langle i_{r_1}, \ldots \langle i_{r_2}, xs\rangle \ldots\rangle \ldots\rangle = \sigma(l)$$
$$\text{in } \langle i_1, \ldots \langle i_{r_2}, \ldots \langle i_{r_1}, xs\rangle \ldots\rangle \ldots\rangle$$

Tensor Expressions

Matrix transposition

```
A = tensor([N1, N2])
B = eq(A, [i1, i2] -> [i2, i1])
```



$$\frac{\mathcal{E}_l[\![\texttt{build}(B)]\!]\sigma_2 = \langle \texttt{i1}, [\langle \texttt{i2}, [\sigma_2(B)]\rangle]\rangle :}{}$$

```
for (int i1 = 0; i1 <= (N1-1); i1++)
 for (int i2 = 0; i2 <= (N2-1); i2++)
  B[i2][i1] = A[i1][i2];
```

Compositions

Contraction

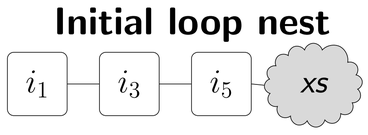
$$\mathcal{P}_{stmt}[\![t' = \texttt{contract}(t_0, t_1, [r_0, r_1])]\!] = \mathcal{P}_{prog}\left[\!\!\left[\begin{array}{l} t_2 = \texttt{vmul}(t_0, t_1, [I, J]) \\ t' = \texttt{add}(t', t_2, [I', \epsilon] \rightarrow I') \end{array} \right]\!\!\right]$$

where

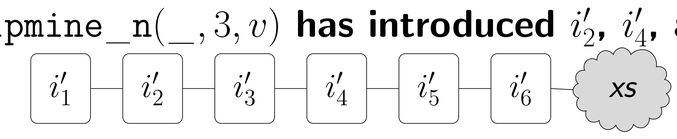
$$I = [\texttt{i0}, \ldots, \texttt{i}(r_0 - 1), \texttt{k}, \texttt{i}(r_0 + 1), \ldots, \texttt{i}s_0]$$
$$J = [\texttt{j0}, \ldots, \texttt{j}(r_1 - 1), \texttt{k}, \texttt{j}(r_1 + 1), \ldots, \texttt{j}s_1]$$
$$I' = (I \backslash \{\texttt{k}\})\ ||\ (J \backslash \{\texttt{k}\})$$

Tiling

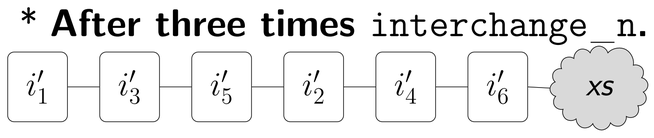
Initial loop nest



`stripmine_n`$(\_, 3, v)$ **has introduced** i'_2**,** i'_4**, and** i'_6

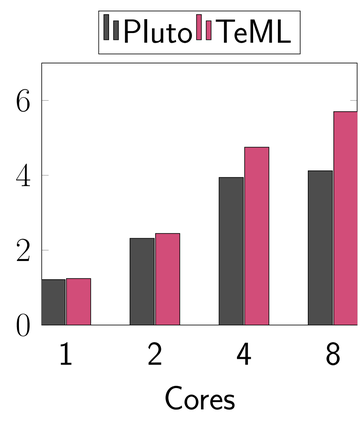


*** After three times** `interchange_n`**.**



Experiments

Helmholtz



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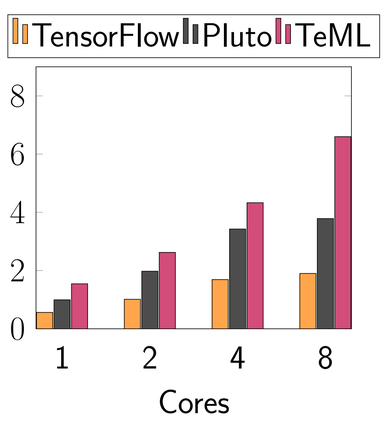
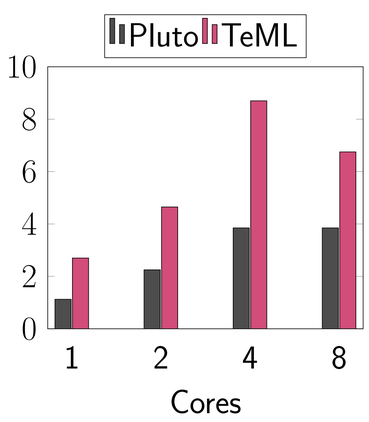


Grouped convolutions



- On Intel(R) Core(TM) i7-4910MQ CPU (2.90GHz, 8 hyperthreads, 8192KB of shared L3 cache), Ubuntu 16.04.
- Generated C programs compiled with the Intel C compiler ICC 18.02 (flags: -O3 -xHost -qopenmp)

With TeML:

- Pluto (v 0.11.4) optimizations reproducible
- Capability to express better optimization paths

Future Work

- Abstractions for memory virtualization, stencil patterns, sparse tensors and corresponding semantics, extensions for parallelism support, type system

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