Implementing an Optimized Double Negation Translation in Dedukti

Internship Proposal

Supervision. Émilie Grienenberger (`emilie.grienenberger@ens-paris-saclay.fr`), Olivier Hermant (`olivier.hermant@mines-paristech.fr`).
Location. LMF, ENS Paris-Saclay and Centre de recherche en informatique, Mines Paris (site de Fontainebleau).

Context. Double-negation translations and their extensions, e.g. A-translation, are used to translate proofs from classical logics, that support the excluded-middle law, to constructive logics. One of the challenges is, when turning a classical proof of *Theorem* $\mathcal{T}$ into a constructive proof of *Theorem* $|\mathcal{T}|^{\neg}$,to preserve as much as possible the equality $|\mathcal{T}|^{\neg} = \mathcal{T}$.

To this aim, aggressive optimizations of the number of negations introduced are necessary.

- by Fréderic Gilbert [Gil15], with an optimality proof,
- by Gilles Dowek [Dow16] (and further improvements [BH13]), where the ocurrences of connectives may be individually chosen to be classical or intuitionistic,
- appealing to focusing techniques [LM13].

Dedukti is a proof checker that is primarily geared towards proof-assistant interoperability, making effectively possible sharing among different assistants, like Coq and Isabelle/HOL. In order to make classical proofs interoperable with constructive systems, it has implemented an oecumenical logic [BDG+21], that mixes classicism and constructivism, where each connective can be chosen to be of classical or constructive nature.

Objectives. The goal of this internship is to improve the double-negation translations of Dedukti, by following the line defined by Gilbert. This will require a theoretical study, in particular the impact of such translations on oecumenical logic, an implementation, and will lead to an increase of the number theorems and proofs that are of purely constructive nature, eventually enabling greater interoperability.

Prerequisites. This internship is mostly self-contained and, besides interest in Computer Science, requires:

- notions of theoretical Computer Science or a good mathematical backgound;
- knowledge of a functional programming language.

References

[BDG$^+$21] Frédéric Blanqui, Gilles Dowek, Émilie Grienenberger, Gabriel Hondet, and François Thiré. Some axioms for mathematics. In Naoki Kobayashi, editor, *6th International Conference on Formal Structures for Computation and Deduction, FSCD 2021, July 17-24, 2021, Buenos Aires, Argentina (Virtual Conference)*, volume 195 of *LIPIcs*, pages 20:1–20:19. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2021.

[BH13] Mélanie Boudard and Olivier Hermant. Polarizing double-negation translations. In Ken McMillan, Aart Middeldorp, and Andreï Voronkov, editors, *LPAR*, volume 8312 of *LNCS ARCoSS*, pages 182–197. Springer, 2013.

[Dow16] Gilles Dowek. On the definition of the classical connectives and quantifiers. *CoRR*, abs/1601.01782, 2016.

[Gil15] Frédéric Gilbert. A lightweight double-negation translation. In Ansgar Fehnker, Annabelle McIver, Geoff Sutcliffe, and Andrei Voronkov, editors, *20th International Conferences on Logic for Programming, Artificial Intelligence and Reasoning - Short Presentations, LPAR 2015, Suva, Fiji, November 24-28, 2015.*, volume 35 of *EPiC Series in Computing*, pages 81–93. EasyChair, 2015.

[LM13] Chuck Liang and Dale Miller. Kripke semantics and proof systems for combining intuitionistic logic and classical logic. *Ann. Pure Appl. Log.*, 164(2):86–111, 2013.