KWARC: Knowledge Adaptation and Reasoning for Content

Research in Foundations, Interaction, and Semantization

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The KWARC Group

Overview: KWARC Research and Projects

Applications: eMath 3.0, Active Documents, Semantic Spreadsheets, Semantic Help Systems, Semantic CAD/CAM, Change Mangagement, ...

Foundations of Math:

- MathML, `OpenMath`
- advanced Type Theories
- MMT: Modular Math Theories
- Logic Morphisms/Atlas
- Theorem Prover Interoperability

Knowledge Mgt. & Interaction:

- Semantic Interpretation
- JOBAD: Document-Embedded Interaction
- TNTBase: Versioned XML Storage
- Math Archives

Semantization:

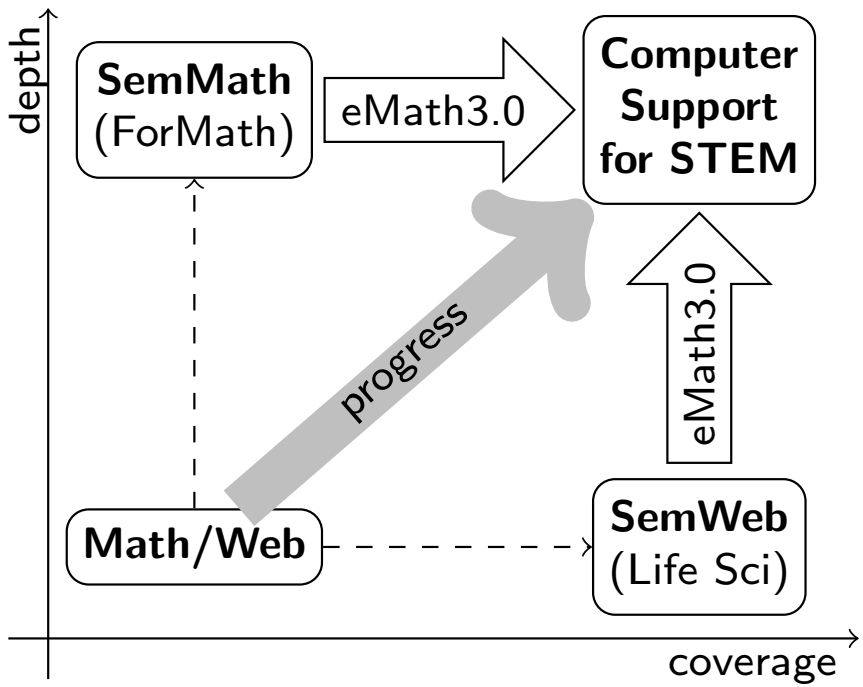
- LaTeXML: LaTeX $\rightarrow$ XML
- sTeX: Semantic LaTeX
- invasive editors
- Context-Aware IDEs
- Mathematical Corpora
- Linguistics of Math

Foundations: Computational Logic, Web Technologies, OMDoc

Contributions from KWARC@Jacobs@Bremen

- ⚠ STEM Knowledge: more like a Digital Library than the Open WWW ⚠
  (reviewed publication $\rightsquigarrow$ less junk, little duplication, partly inaccessible)

- Combination of SemMath and SemWeb
- Expertise in Semantics of STEM Docs
- Expressive Analysis Target Format (OMDoc)
- Software Stack for Semantic Processing
- eSTEM3.0 System Planetary (Active Docs)
- Invasive authoring (Office/LaTeX)
- Semantic Analysis for LaTeX-based Corpora
  (arXiv, ZBL, PlanetMath...)



- ⚠ We use Math as a test tube for STEM (Science, Tech, Eng, & Math) ⚠

KWARC People I

- Michael Kohlhase (Professor; Project Lead)
  - Projects: OMDoc, sTEX, arXMLiv, NL Semantics, MathSearch...
  - Specializes: "world domination" (ubiquitous computer-supp. math)

- Heinrich Stamerjohanns (Director of CS Labs; Senior Collaborator)
  - Projects: arXMLiv, PhysML, PhysNet, JGrader, ...
  - Specialties: Web Information Systems, Document Management...

- Florian Rabe (Jacobs PostDoc)
  - Projects: OMDoc2, LATIN, MMT (loves category theory)
  - Specializes: metalogics, language design, math foundations, ...

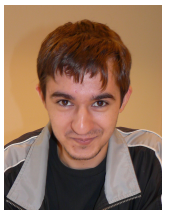
- Andrea Kohlhase (Jacobs PostDoc)
  - Projects: SiSsi, Planetary, MathSearch
  - Specializes: Semantic Interaction, Semantic Design, HCI

- Fulya Horozal (Doctoral Student (Metalogics))
  - Projects: LATIN
  - Thesis: Representing Logics and Logic Morphisms

KWARC People II

- Constantin Jucovschi (Doctoral Student (Editing Support))
  - Projects: SiSsi, PLANETARY
  - Thesis: Integrated Development Environemtns for STEM Documents

- Deyan Ginev (Doctoral Student (Math Linguistics))
  - Projects: LaMaPuN, `arXMLiv`, PLANETARY, sTEX, ...
  - Thesis: Semantizising Math Formulae



- Michnea Iancu (Doctoral Student (OMDoc2))
  - Projects: MathSearch
  - Thesis: informal MMT, OAFF
- M.Sc. Students: with thesis titles and ETA
  - Aivaras Jacubauskas: *Declarative Representation of Computation* (2013)
  - Corneliu Prodescu: *Formula Search Engines* (2014)
- B.Sc. Students Jan Dörrie, Felix Mance, Stefan Mirea, Daniel Rachev, Dharam Kapila (do thesis research and help with the KWARC projects)

Introduction/Background

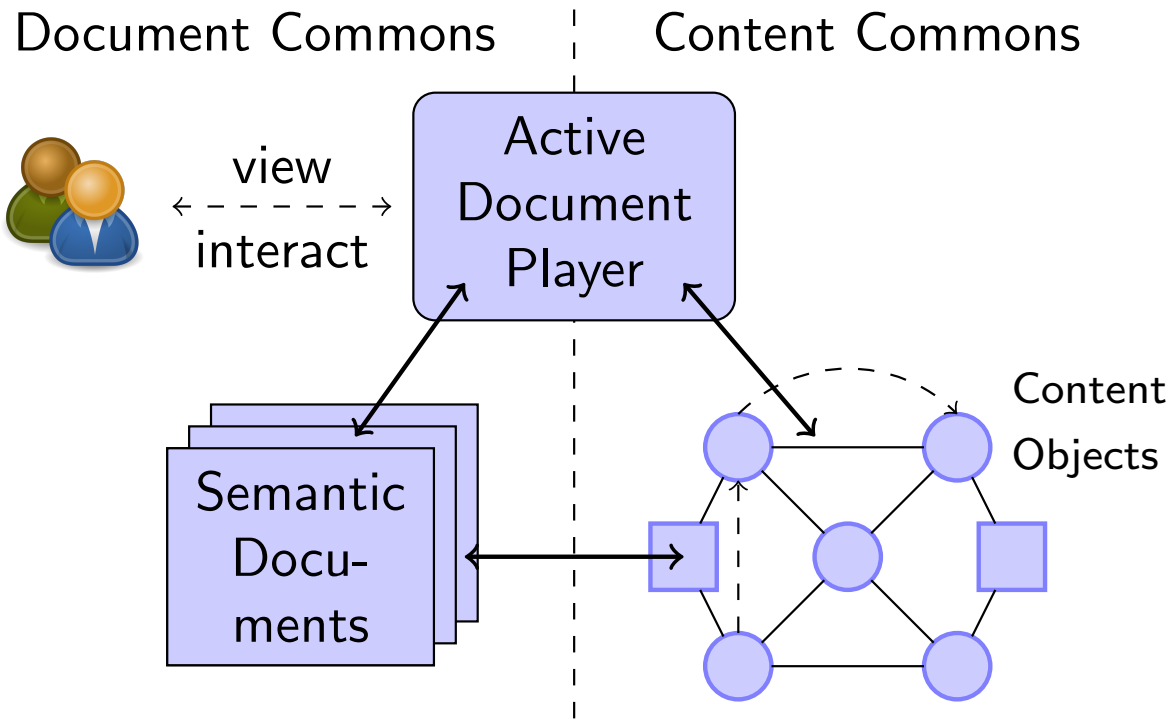
- Mathematics plays a fundamental role in Science, Technology, and Engineering (learn from Math, apply for STEM)
- Mathematical knowledge is rich in content, sophisticated in structure, and technical in presentation,
- its conservation, dissemination, and utilization constitutes a challenge for the community and an attractive line of inquiry.
- Challenge: How can/should we do mathematics in the 21st century?
- Mathematical knowledge and objects are transported by documents
- Three levels of electronic documents:
  1. digitized (usually from print):
  2. presentational: encoded text interspersed with presentation markup
  3. semantic: encoded text with functional markup for the meaning

  transforming down is simple, transforming up needs humans or AI.
- Observation: Computer support for access, aggregation, and application is (largely) restricted to the semantic level.
- This talk: How do we do maths and math documents at the semantic level?

Semantic Interaction with Technical Documents

The Active Documents Paradigm

- **Definition 1** The active documents paradigm (ADP) consists of
  - *semantically annotated documents* together with
  - background ontologies (which we call the content commons),
  - *semantic services* that use this information
  - a document player application that embeds services to make documents executable.



- **Example 2** Services can be program (fragment) execution, computation, visualization, navigation, information aggregation and information retrieval

What you get out of it depends on what you put into it

- The level of support depends on the level of semantic enhancement!
- Presentation Structure Level: e.g. legacy-rich sites like arXiv.org and PlanetMath.org (or ScienceDirect.com)
- Semantic Level: e.g. flexiformal digital libraries: Course Notes and Manual
- Formal Level: e.g. specification and verification in the LATIN Logic Atlas

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  - discourse-level/formula folding
  - icon menu for embedded services,
  - localized discussions + notification bar (with Math)
  - localized refereeing
  - formula search engine
  - Ontology-based access (here: MSC)
- Semantic Level: e.g. flexiformal digital libraries: Course Notes and Manual
- Formal Level: e.g. specification and verification in the LATIN Logic Atlas

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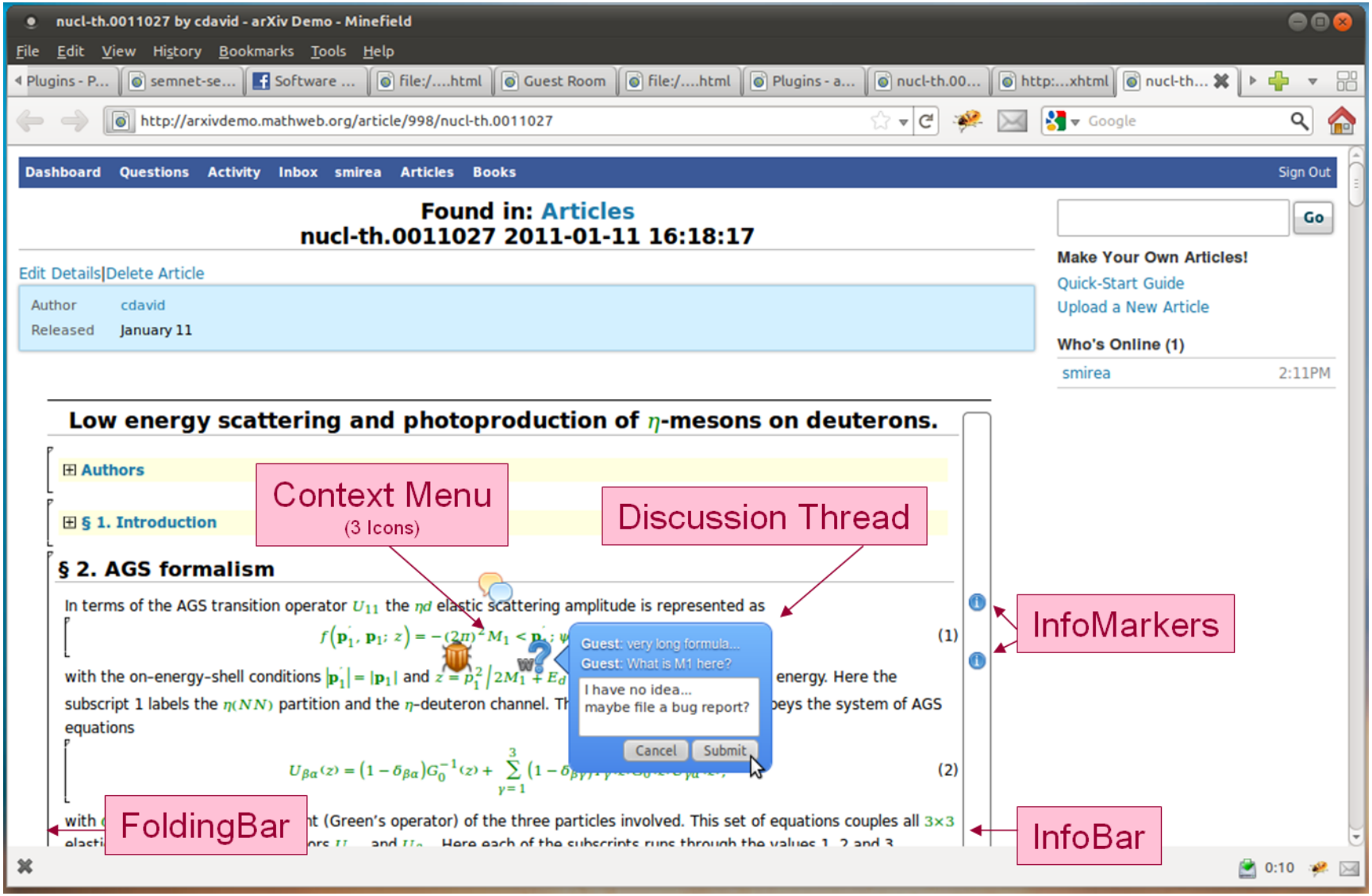

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- Semantic Level: e.g. flexiformal digital libraries: Course Notes and Manual
  - prerequisites graph and guided tours
  - definition lookup
  - unit conversion
  - "monographs", that aggregate "modules" into narrative structures
  - semantic editing facilities
  - Management of Change (versioning and change impact analysis)
- Formal Level: e.g. specification and verification in the LATIN Logic Atlas

What you get out of it depends on what you put into it

Definition Lookup

$f \subseteq X \times Y$, is called a **partial function** , iff for all $x \in X$ there is at

Definition Lookup Results

DEFINITION:

Cartesian product :
$A \times B := \{\langle a, b\rangle \mid a \in A \wedge b \in B\}$, call $\langle a, b\rangle$ **pair** .

om

Semantic Folding

$s = s_i + v_i \Delta t + \frac{1}{2} a (\Delta t)^2$

Fold

Semantic Fold

$s = s_i + s_v + s$

contribution from acceleration

Unit Conversion

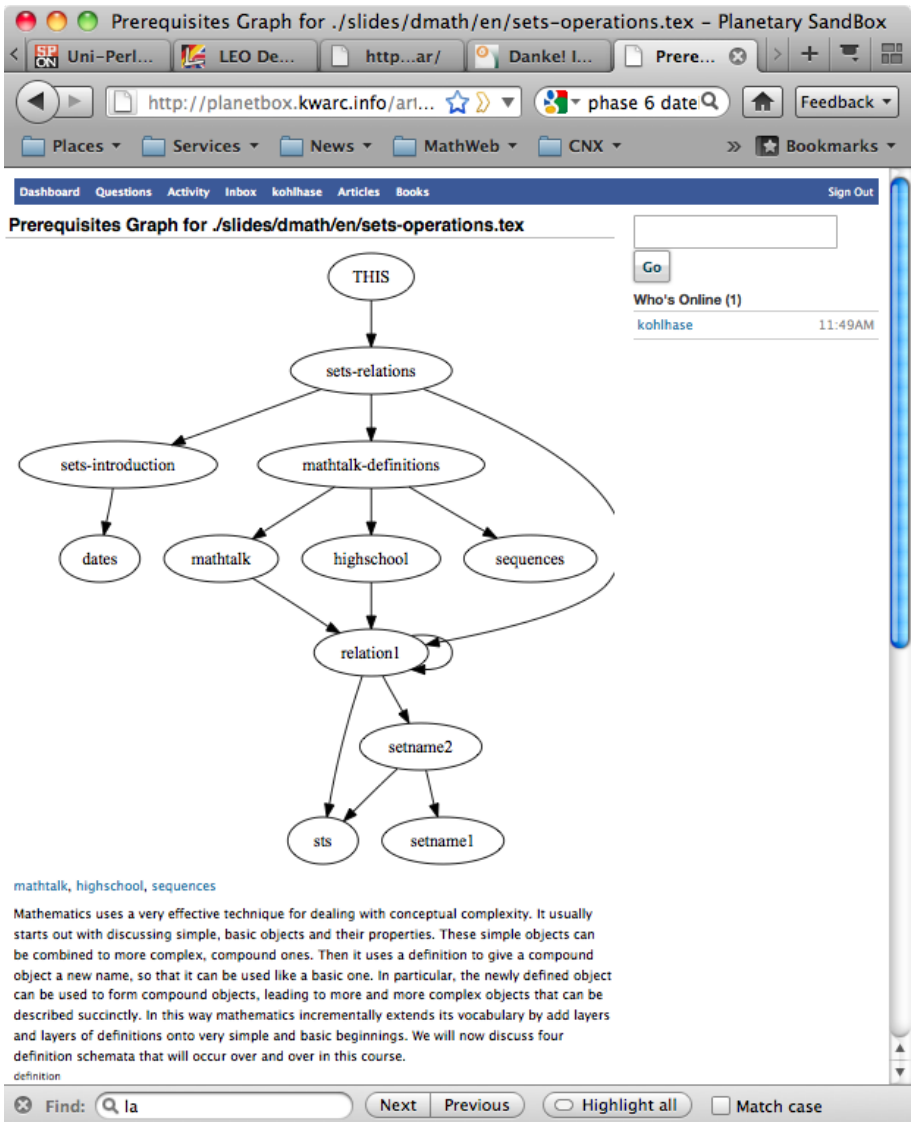
City A is 9144ft from city B and 5164ft from city C .

Look-up Definition
convert to miles
convert to meters
convert to feet
convert to inches
convert to yards

convert the units

City A is 3048m from city B and 5164ft from city C .

Prerequisites Navigation



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- Semantic Level: e.g. flexiformal digital libraries: Course Notes and Manual
- Formal Level: e.g. specification and verification in the LATIN Logic Atlas
  - flexible elision of brackets
  - argument reconstruction via external TWELF system
  - verification by the HETS system

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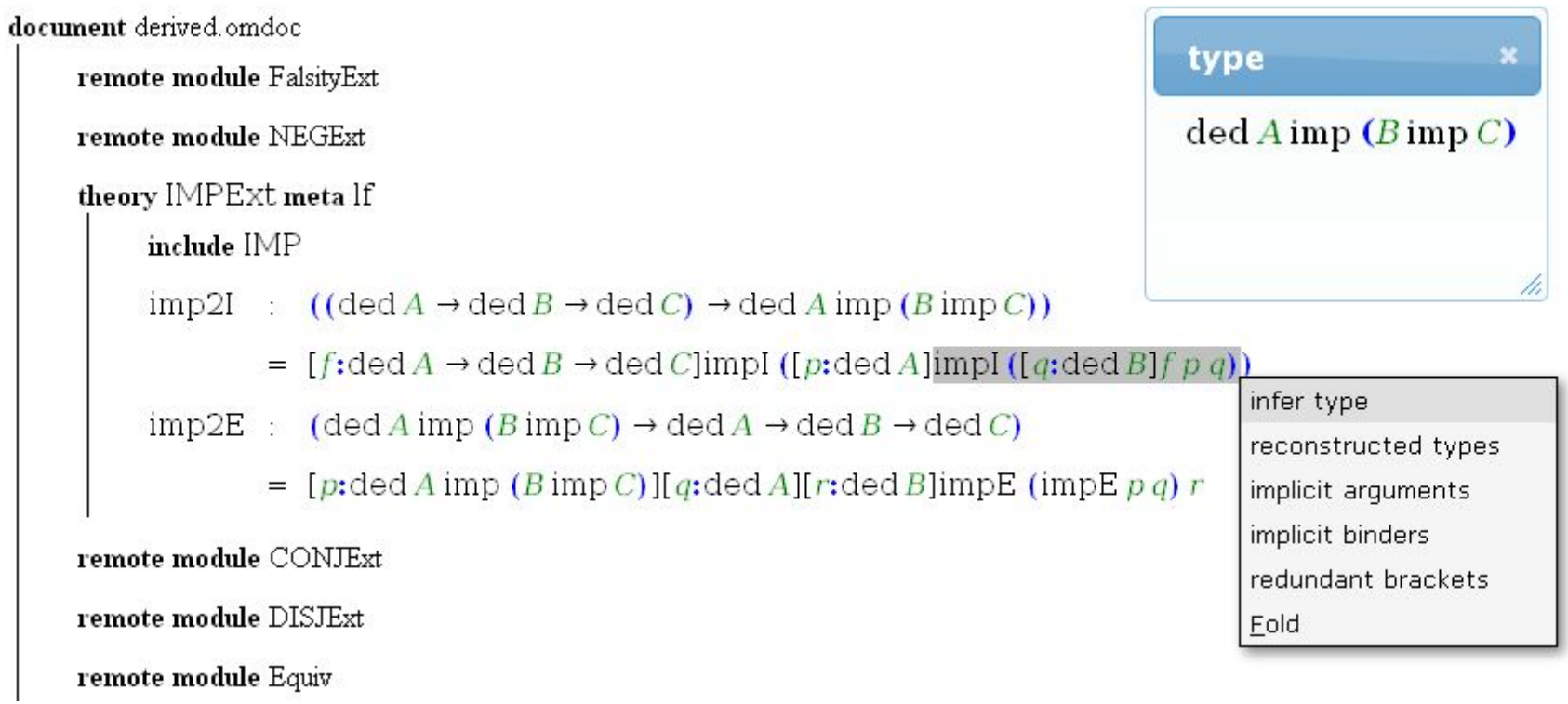
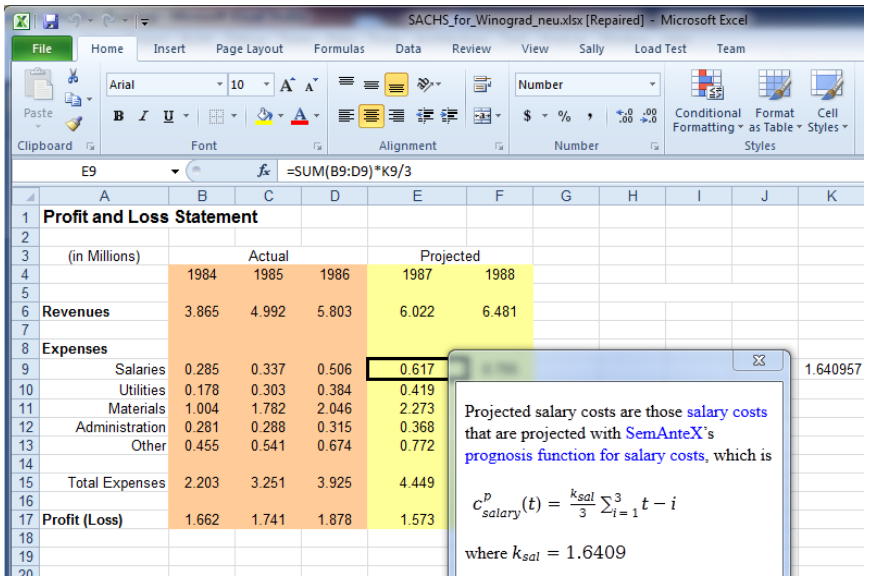
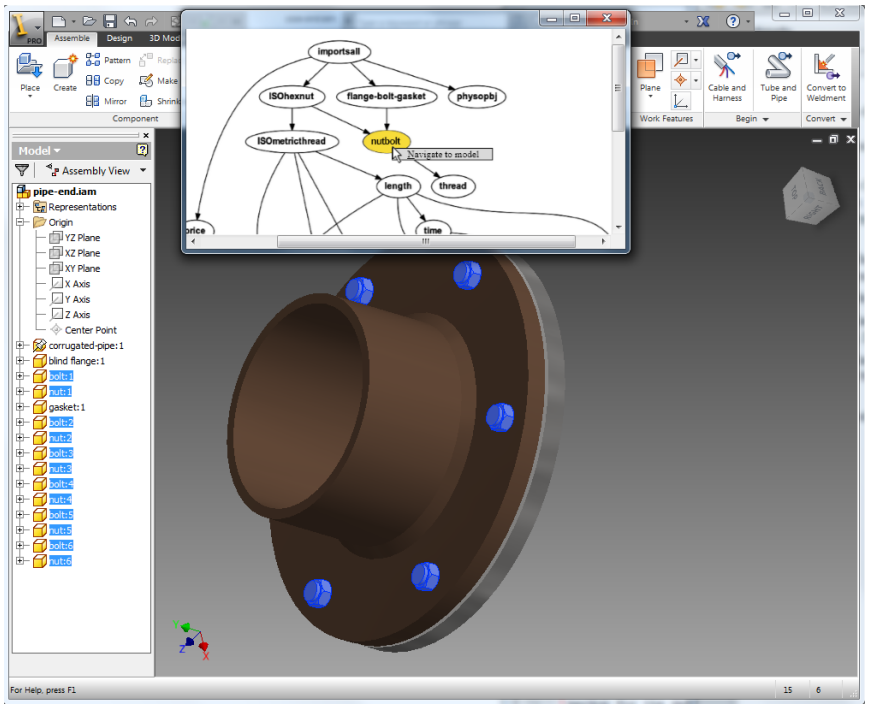


Embedding Semantic Services into Proprietary Applications

Definition Lookup in Excel



Semantic Navigation in a CAD System



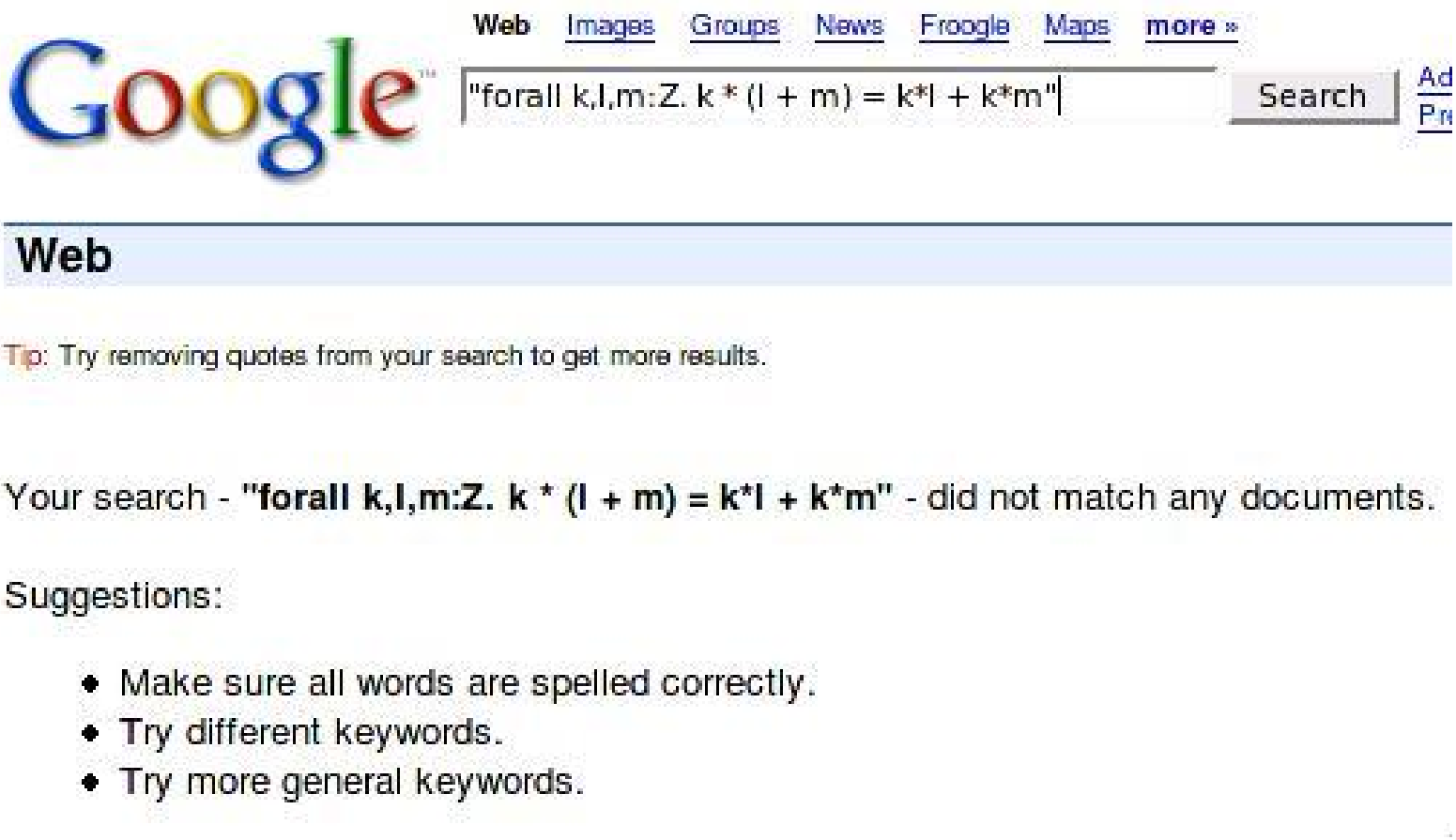
And of course. . . Math Search!

More Mathematics on the Web

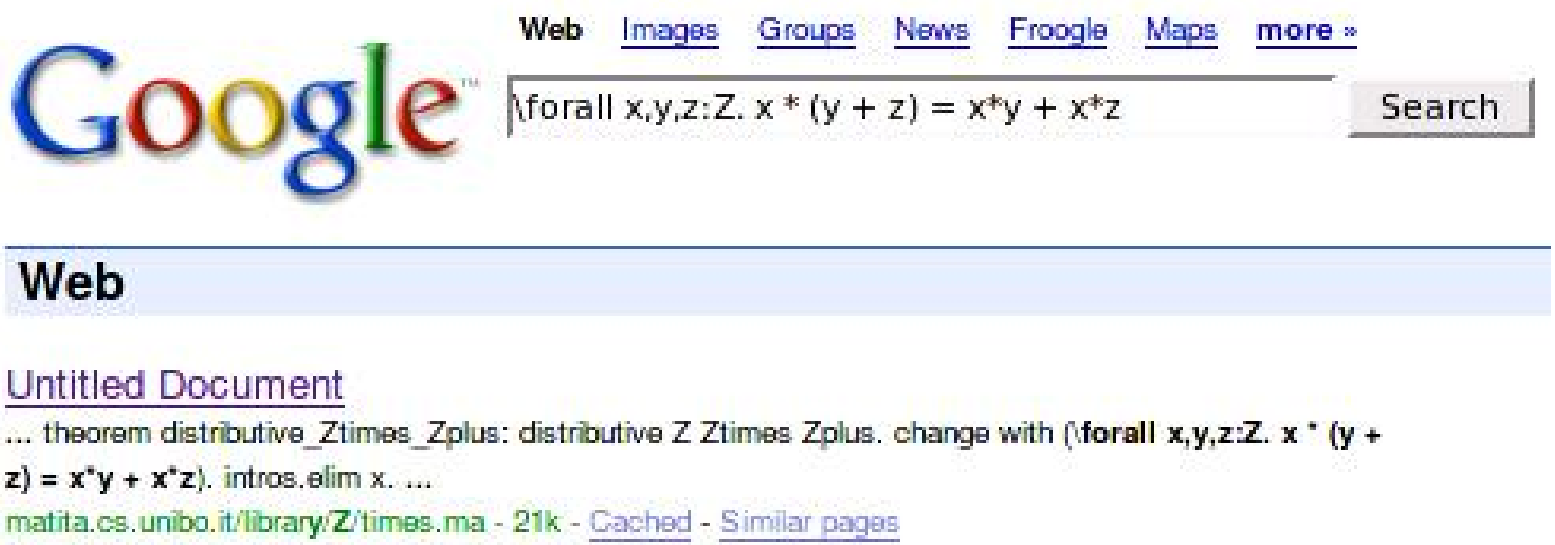
- The Connexions project (http://cnx.org)
- Wolfram Inc. (http://functions.wolfram.com)
- Eric Weisstein's MathWorld (http://mathworld.wolfram.com)
- Digital Library of Mathematical Functions (http://dlmf.nist.gov)
- Cornell ePrint arXiv (http://www.arxiv.org)
- Zentralblatt Math (http://www.zentralblatt-math.org)
- ...
- Question: How will we find content that is relevant to our needs
- Idea: try Google (like we always do)
- Scenario: Try finding the distributivity property for $\mathbb{Z}$

$$(\forall k, l, m \in \mathbb{Z}.k \cdot (l + m) = (k \cdot l) + (k + m))$$

Searching for Distributivity



Searching for Distributivity

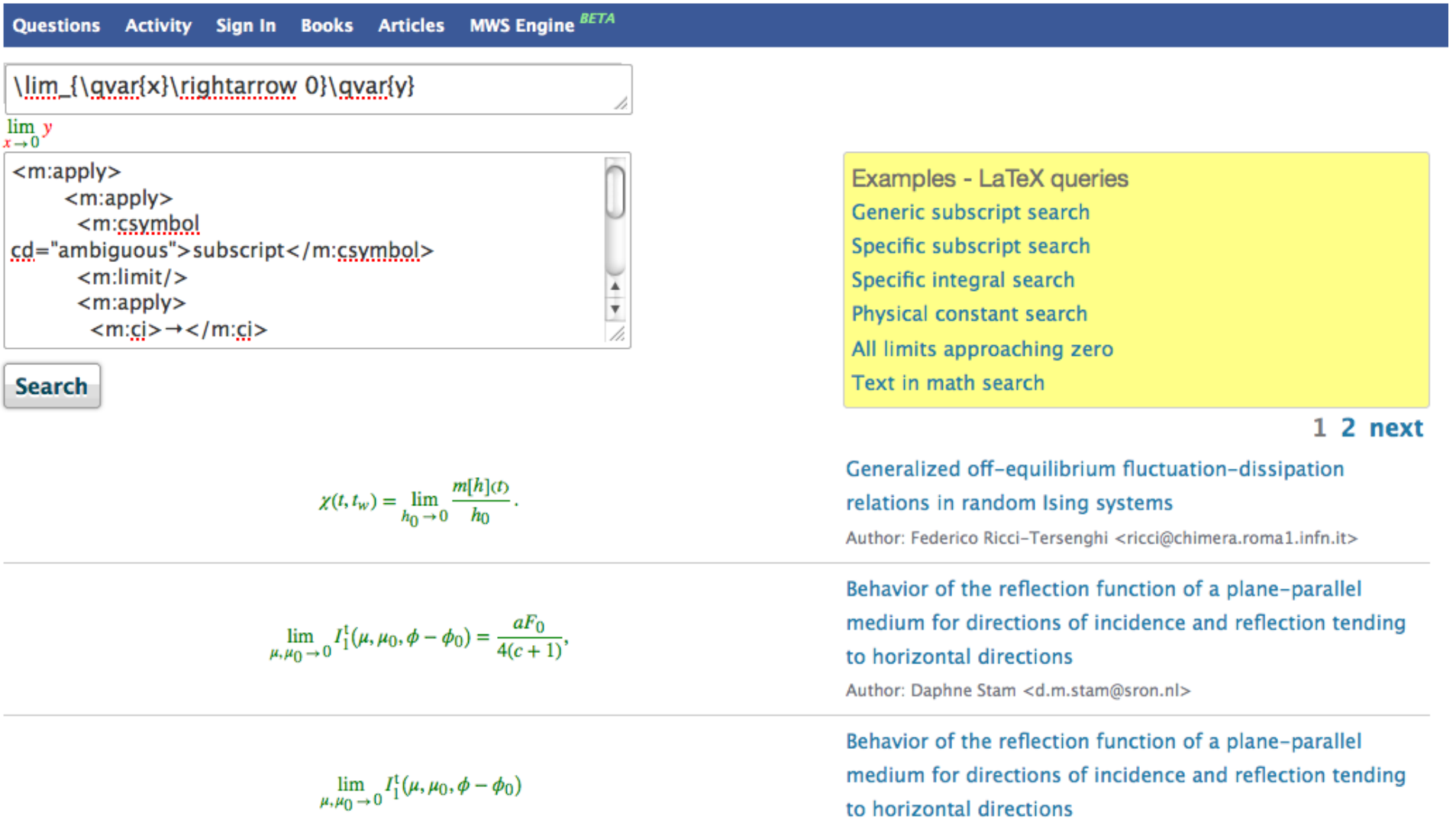


Searching for Distributivity

Of course Google cannot work out of the box

- Formulae are not words:
  - a, b, c, k, l, m, x, y, and z are (bound) variables.
    (do not behave like words/symbols)
  - where are the word boundaries for "bag-of-words" methods?
- Idea: Need a special treatment for formulae (translate into "special words")
  Indeed this is done ([MY03, MM06, LM06, MG11])
  ...and works surprisingly well (using Lucene as an indexing engine)
- Idea: Use database techniques (extract metadata and index it)
  Indeed this is done for the `Coq`/`HELM` corpus ([AGC+06])
- Our Idea: Use Automated Reasoning Techniques
  (free term indexing from theorem prover jails)

Instead of a Demo: LaTeX-based Search on the arXiv



Generalization Queries

- Application: Find (possibly) applicable theorems
- **Example 3** A researcher wants to estimate $\int_{\mathbb{R}^2} |\sin(t)\cos(t)|dt$ from above
  - Idea: Find inequation such that $\int_{\mathbb{R}^2} |\sin(t)\cos(t)|dt$ matches left hand side.
  - Query: $\int_{\mathbb{R}^2} |\sin(x)\cos(x)|\,dx \leq \boxed{rhs}$
  - matches e.g. Hölder's Inequality in the index: ($\boxed{i}$ are universal variables)

$$\int_{\boxed{D}} \left|\boxed{f}(x)\boxed{g}(x)\right| dx \leq \left(\int_{\boxed{D}} \left|\boxed{f}(x)\right|^p dx\right)^{\frac{1}{p}} \left(\int_{\boxed{D}} \left|\boxed{g}(x)\right|^q dx\right)^{\frac{1}{q}}$$

  - Solution: Instantiate query accordingly and get

$$\int_{\mathbb{R}^2} |\sin(x)\cos(x)|\,dx \leq \left(\int_{\mathbb{R}^2} |\sin(x)|^p\,dx\right)^{\frac{1}{p}} \left(\int_{\mathbb{R}^2} |\cos(x)|^q\,dx\right)^{\frac{1}{q}}$$

Problem: Where do the index formulae come from in particular the universal variables (we'll come back to that later)

Instead of a Demo: Applicable Theorem Search in Mizar

```
definition
 let k, n be Ordinal;
 pred k divides n means :Def3: :: MTEST1:def 3
 ex a being Ordinal st n = k *^ a;
 reflexivity
 proof
  let n be Ordinal; :: thesis:
  thus ex a being Ordinal st n = n *^ a ;
```

ATP Proof not found

status: Timeout
Suggest hints, Unification query,

Suggested hints

t73_card_2, t39_ordinal2,

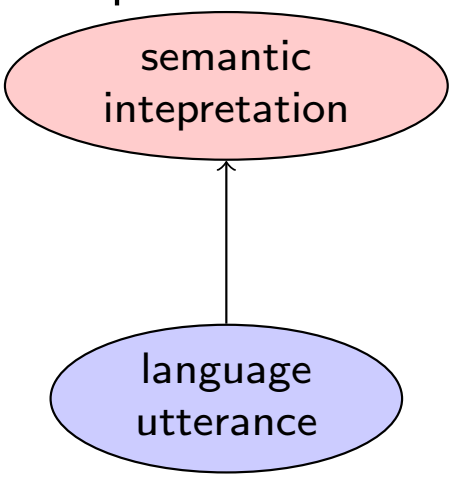
Try SPASS, Export problem to SystemOnTPTP

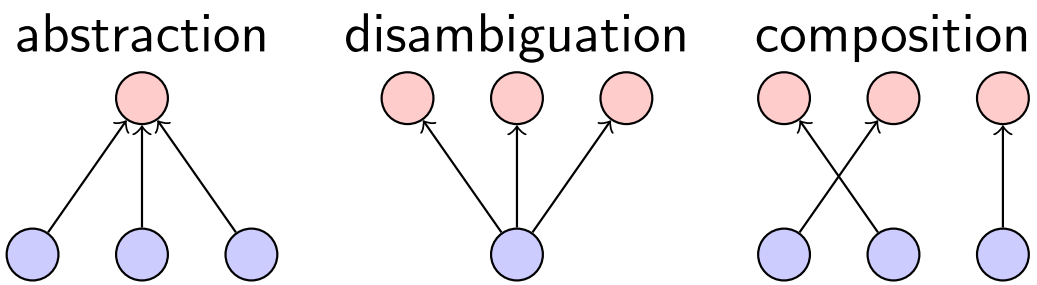
```
  :: thesis:
 end;
end;
```

Towards Semantic Math Libraries: Semantization

Language and Information

- humans use words (sentences, texts) in natural languages to represent information
- but:
- what really counts is not the words themselves, but the meaning information they carry.
- for questions/answers, it would be very useful to find out what words (sentences/texts) mean.
- Interpretation of natural language utterances: three problems





The Reconstruction Problem

- Mathematical communication relies on the inferential capability of the reader.
- semantically relevant arguments are left out (or ambiguous) to save notational overload (reader must disambiguate or fill in details.)

$$\log_2(x) \text{ vs. } \log(x) \qquad [\![\mathbf{A}]\!]^{\mathcal{M}} \text{ \_phi vs. } [\![\mathbf{A}]\!]$$

- condensed notation: $f(x+1)\pm 2\pi = g(x-1)\mp 2i$ (stands for 2 equations)
- ad hoc extensions: $\#(A \cup B) \leq \#A + \#B$ (exceptions for ∞)
- overt ambiguity: $\sin x/y$ vs. $\frac{\sin x}{y}$ vs. $\sin \frac{x}{y}$ vs. $-1 \leq \sin x/\pi \leq 1$
- size of the gaps varies with the intended readership and the space constraints.
- can be so substantial, that only a few specialists in the field can understand

The Notation/Context Problem

- idiosyncratic notations that are introduced, extended, discarded on the fly

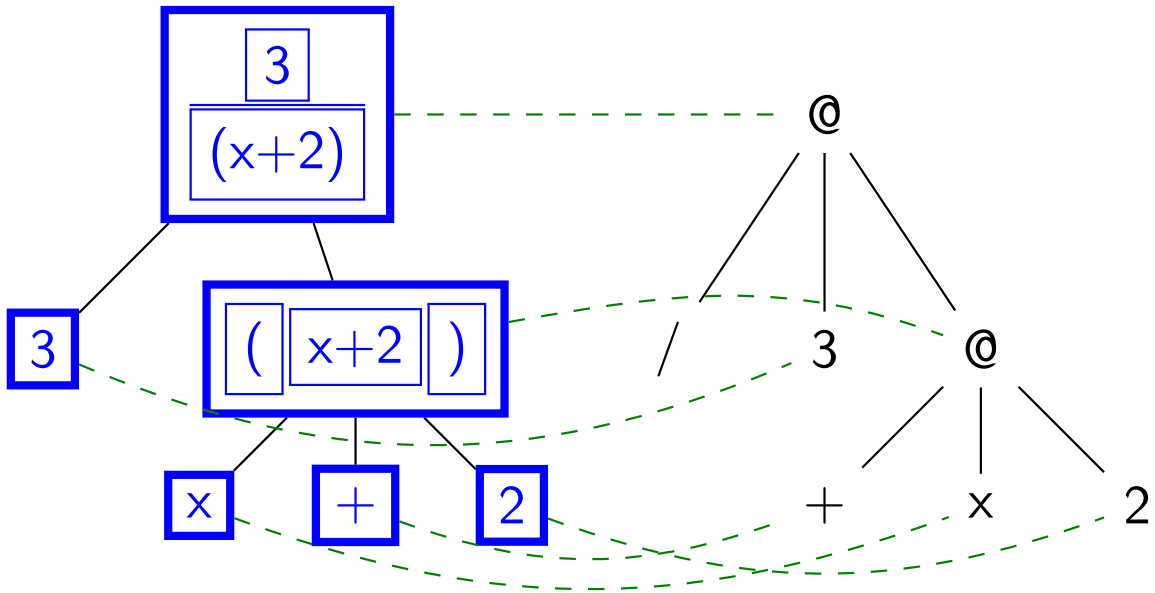
$$\lambda X_\alpha.X =_\alpha \lambda Y_\alpha.Y \hat{=} \mathbf{I}^\alpha$$

  meaning of α depends on context: object type vs. mnemonic vs. type label.
- even "standard notations" depend on the context, e.g. binomial coefficients: $\binom{n}{k}$, ${}_nC^k$, C^n_k, and C^k_n all mean the same thing: $\frac{n!}{k!(n-k)!}$ (cultural context)
- Notation scoping follows complex rules (notations must be introduced)
  - *We will write $\wp(S)$ for the set of subsets of S* (for the rest of the doc)
  - *We use the notation of [BrHa86], with the exception...* (by reference)
  - *Let S be a set and $f : S \to S$...* (scope local in definition)
  - *where w is the...* (scope local in preceding formula)
  - Book on group theory in Bourbaki series uses notation [Bou: Algebra]

Observation: Notation scoping is different from the one offered by TEX/LATEX

Parallel Markup e.g. in MathML I

- Idea: Combine the presentation and content markup and cross-reference

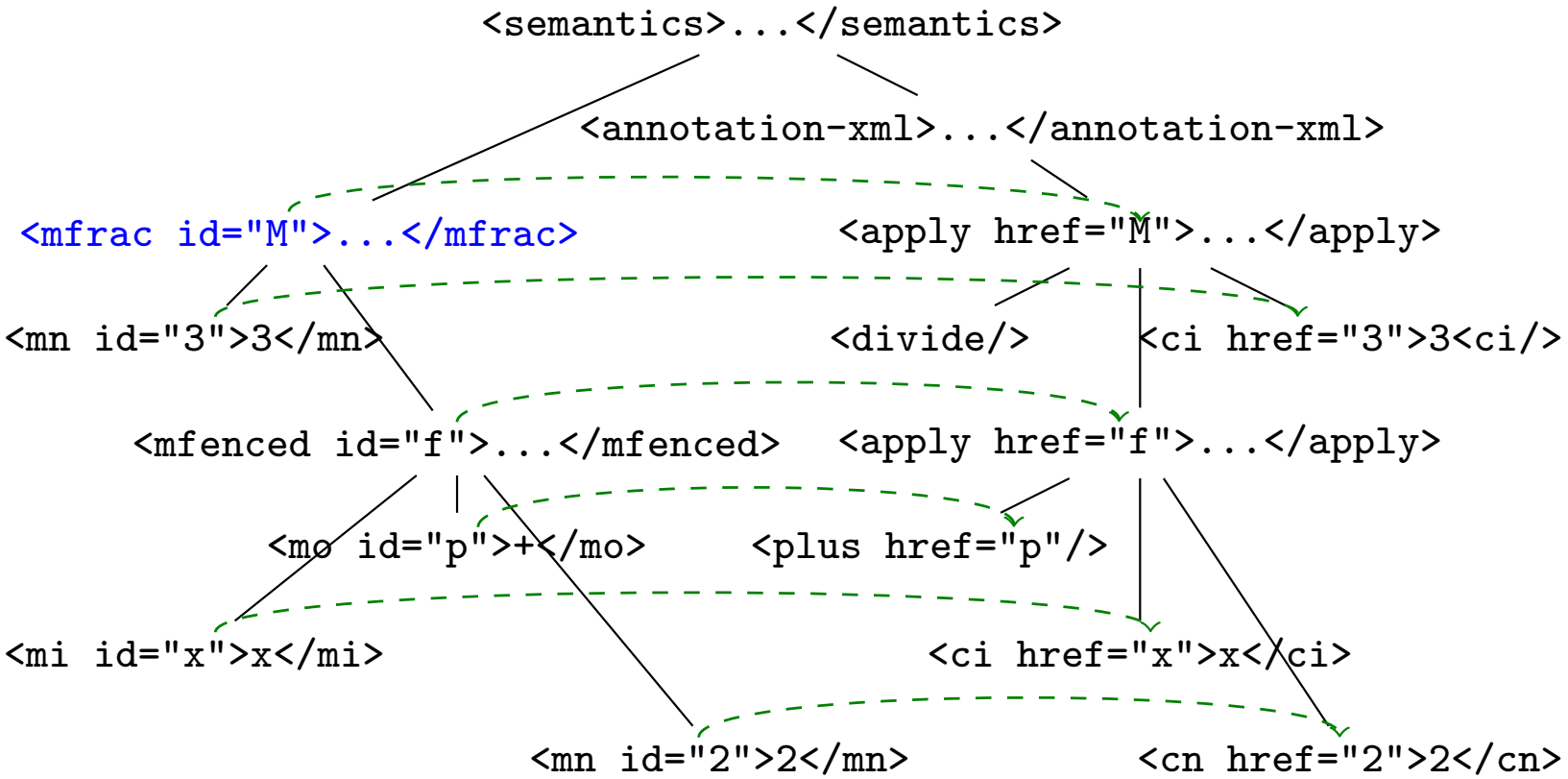


- use e.g. for semantic copy and paste.

(click on presentation, follow link and copy content)

Parallel Markup e.g. in MathML II

- Concrete Realization in MathML: `semantics` element with presentation as first child and content in `annotation-xml` child



STEM Corpora and Semantics Recovery

- Corpora at the presentation/structure level
  - The arXMLiv Corpus: arxmliv ca. 800k LaTeX documents from STEM discplines.
  - The ZBMath Corpus: ca. 3.3M abstracts/reviews for math articles in LaTeX.
  - PlanetMath.org: ca. 9k math encyclopedia articles in Planetary.
- Corpora at the content level
  - OpenMath/MathML Content Dictionaries: ca. 100 OpenMath documents designed as targets for semantic annotations.
  - The STC Corpus: ca. 4k semantically annotated course modules/problems in Planetary.
- Formal Corpora
  - The Mizar Corpus: ca. 100k formalized statements/proofs in Planetary.
  - The LATIN Logic Atlas: Meta-logical development of mathematical foundations.

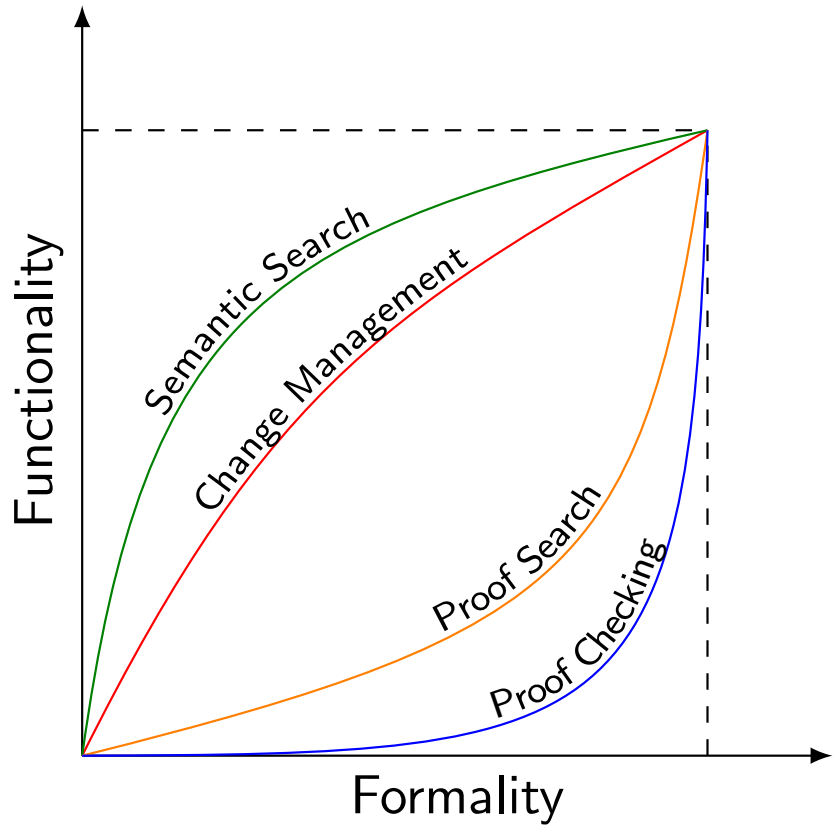
The Flexiformalist Program

The Flexiformalist Program (Details in [Koh13])

- The development of a regime of partially formalizing
  - mathematical knowledge into a modular ontology of mathematical theories (content commons), and
  - mathematical documents by semantic annotations and links into the content commons (semantic documents),
- The establishment of a software infrastructure with
  - a distributed network of archives that manage the content commons and collections of semantic documents,
  - semantic web services that perform tasks to support current and future mathematic practices
  - active document players that present semantic documents to readers and give access to respective
- the re-development of comprehensive part of mathematical knowledge and the mathematical documents that carries it into a flexiformal digital library of mathematics.

Flexible Levels of Formality/Semantics

- We are developing regimes of flexibly adding semantics (partial formalization)
- semantic services have distinct formalization thresholds
- start with the low-hanging fruit (80-20)



- Support Math Research by "global" services that cross the one-brain barrier
  - e.g. automatically finding representation theorems that transport theorems

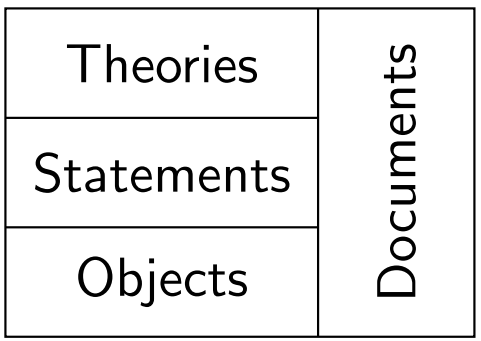
OMDoc: Open Mathematical Documents

OMDoc Philosophy

- OMDoc: Open Mathematical Documents: Representing Knowledge/Documents in Math/STEM.
- General features of knowledge representation languages (primitive/orthogonal?)

Feature	range		
aspect	functional	–	visual/narrative
paradigm	inferential	–	computational
grammar	natural	controlled	extensible
vocabulary	fixed	*vs.*	extensible
foundation	unspecified	fixed	expressible

- Domain Constraint for STEM Documents: Three levels:
  - object/phrase level, e.g. formulae
  - statement/paragraph level, e.g. definition, theorem, proof
  - theory/discourse level, theories, views, models
- OMDoc Philosophy: Interleave all features at all levels via parallel markup (general flexiformalization framework)



OMDoc in a Nutshell (three levels of modeling) [Koh06]

Formula level: OpenMath/C-MathML

- Objects as logical formulae
- symbol meaning by reference to theory level

```
<apply>
 <csymbol cd="ring">plus</c.>
 <csymbol cd="ring">zero</c.>
 <ci>N</ci>
</apply>
```

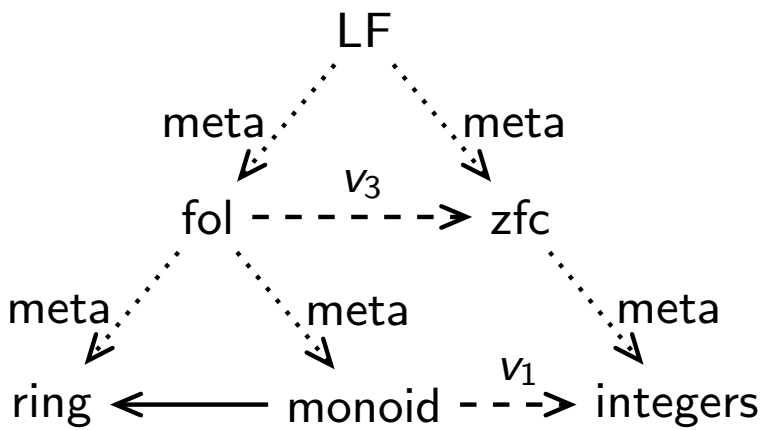
Statement level:

- Definition, Theorem, Proof, Example
- semantics via explicit forms and refs.
- parallel formal & natural language

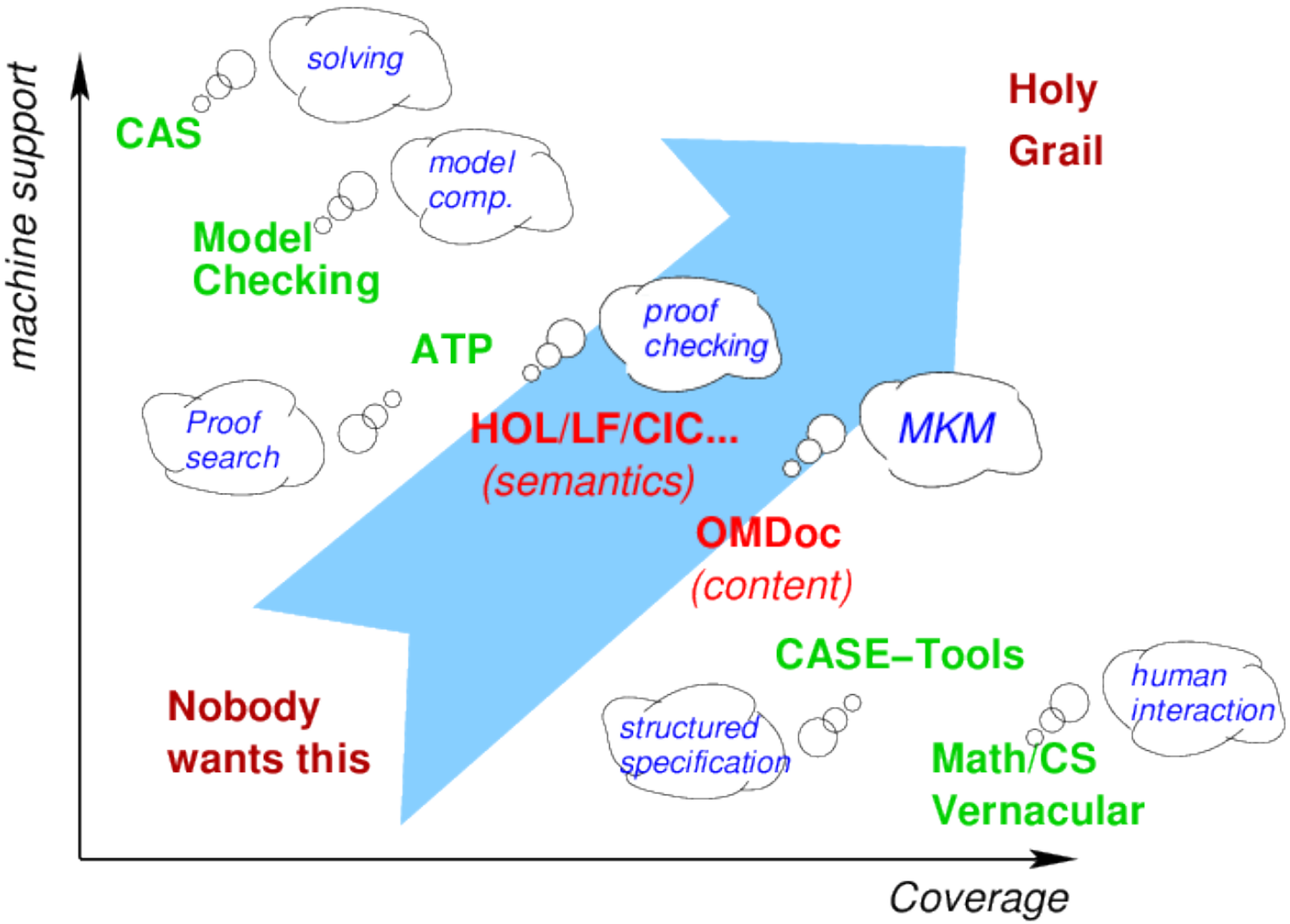
```
<defn for="plus" type="rec">
 <CMP>rec. eq. for plus</CMP>
 <FMP>X + 0 = X</FMP>
 <FMP>X + s(Y) = s(X + Y)</FMP>
</defn>
```

Module level: Theory Graph [RK13]

- inheritance via symbol-mapping
- views by proof-obligations
- logics as meta-theories (logic atlas)
- meta-logics as oracles for type/eq



Situating OMDoc: Math Knowledge Management

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