15-213
"The Class That Gives CMU Its Zip!"

Bits, Bytes, and Integers

Topics

- Representing information as bits
- Bit-level manipulations
  - Boolean algebra
  - Expressing in C
- Representations of Integers
  - Basic properties and operations
  - Implications for C

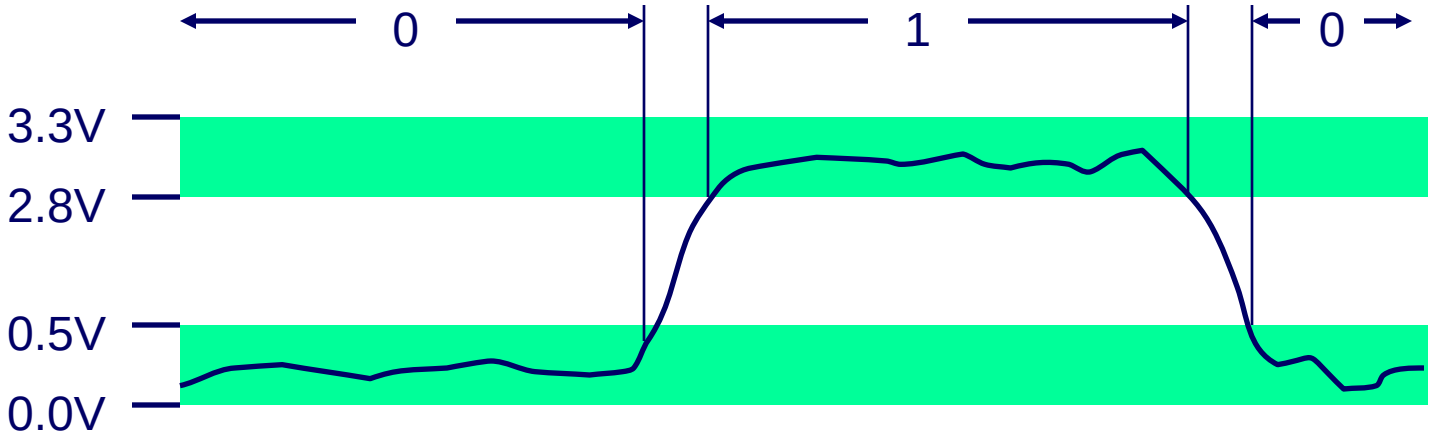
Binary Representations

Base 2 Number Representation

- Represent 15213_{10} as 11101101101101_2
- Represent 1.20_{10} as $1.00110011001100 11[0011]\ldots_2$
- Represent 1.5213×10^4 as $1.1101101101101_2 \times 2^{13}$

Electronic Implementation

- Easy to store with bistable elements
- Reliably transmitted on noisy and inaccurate wires



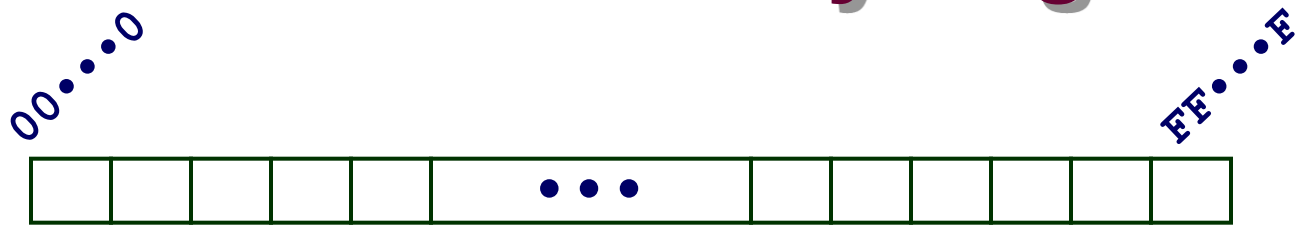
Encoding Byte Values

Byte = 8 bits

- Binary 00000000_2 to 11111111_2
- Decimal: 0_{10} to 255_{10}
  - First digit must not be 0 in C
- Hexadecimal 00_{16} to FF_{16}
  - Base 16 number representation
  - Use characters '0' to '9' and 'A' to 'F'
  - Write $FA1D37B_{16}$ in C as `0xFA1D37B`
    - » Or `0xfa1d37b`

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Byte-Oriented Memory Organization



Programs Refer to Virtual Addresses

- Conceptually very large array of bytes
- Actually implemented with hierarchy of different memory types
- System provides address space private to particular "process"
  - Program being executed
  - Program can clobber its own data, but not that of others

Compiler + Run-Time System Control Allocation

- Where different program objects should be stored
- All allocation within single virtual address space

Machine Words

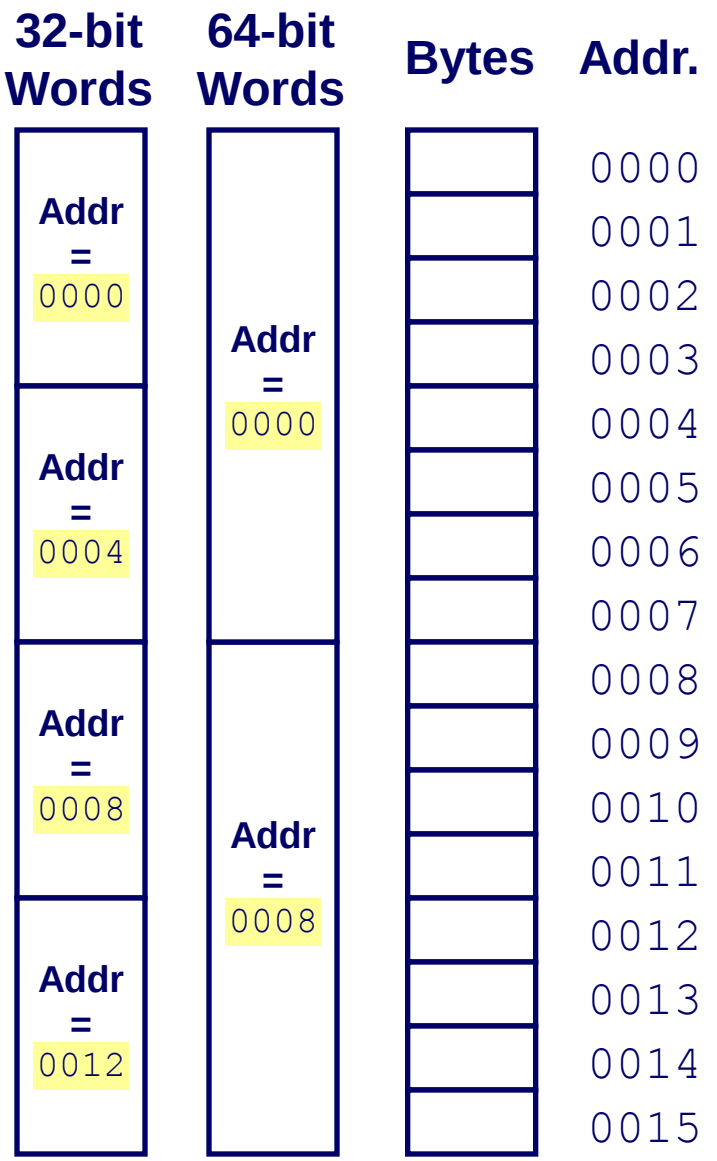
Machine Has "Word Size"

- Nominal size of integer-valued data
  - Including addresses
- Most current machines use 32 bits (4 bytes) words
  - Limits addresses to 4GB
  - Becoming too small for memory-intensive applications
- High-end systems use 64 bits (8 bytes) words
  - Potential address space ≈ 1.8 X 10^{19} bytes
  - x86-64 machines support 48-bit addresses: 256 Terabytes
- Machines support multiple data formats
  - Fractions or multiples of word size
  - Always integral number of bytes

Word-Oriented Memory Organization

Addresses Specify Byte Locations

- Address of first byte in word
- Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Data Representations

Sizes of C Objects (in Bytes)

C Data Type	Typical 32-bit	Intel IA32	x86-64
char	1	1	1
short	2	2	2
int	4	4	4
long	4	4	8
long long	8	8	8
float	4	4	4
double	8	8	8
long double	8	10/12	10/16
char *	4	4	8

- » Or any other pointer

Byte Ordering

How should bytes within multi-byte word be ordered in memory?

Conventions

- Big Endian: Sun, PPC Mac, Internet
  - Least significant byte has highest address
- Little Endian: x86
  - Least significant byte has lowest address

Byte Ordering Example

Big Endian

- Least significant byte has highest address

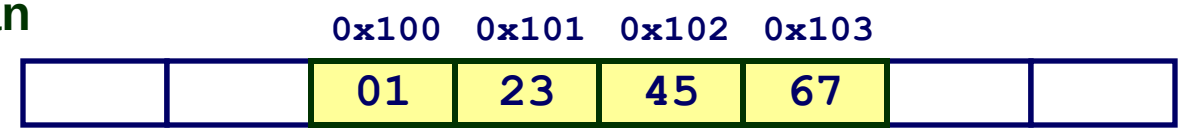
Little Endian

- Least significant byte has lowest address

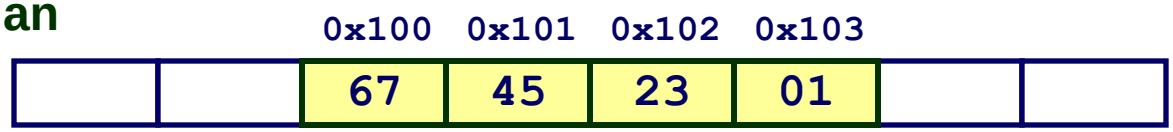
Example

- Variable `x` has 4-byte representation `0x01234567`
- Address given by `&x` is `0x100`

Big Endian



Little Endian



Reading Byte-Reversed Listings

Disassembly

- Text representation of binary machine code
- Generated by program that reads the machine code

Example Fragment

Address	Instruction Code	Assembly Rendition
8048365:	5b	pop %ebx
8048366:	81 c3 ab 12 00 00	add $0x12ab,%ebx
804836c:	83 bb 28 00 00 00 00	cmpl $0x0,0x28(%ebx)

Deciphering Numbers

- Value: 0x12ab
- Pad to 32 bits: 0x000012ab
- Split into bytes: 00 00 12 ab
- Reverse: ab 12 00 00

Examining Data Representations

Code to Print Byte Representation of Data

- Casting pointer to `unsigned char *` creates byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, int len)
{
  int i;
  for (i = 0; i < len; i++)
    printf("0x%p\t0x%.2x\n",
           start+i, start[i]);
  printf("\n");
}
```

Printf directives:

`%p`: Print pointer

`%x`: Print Hexadecimal

show_bytes Execution Example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

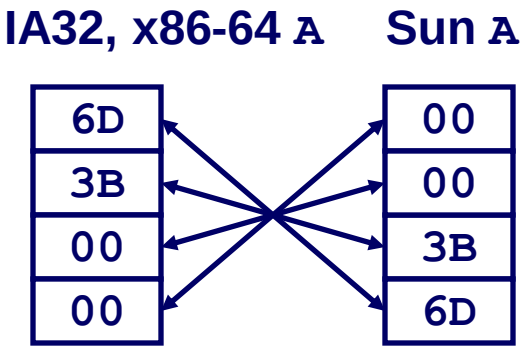
Result (Linux):

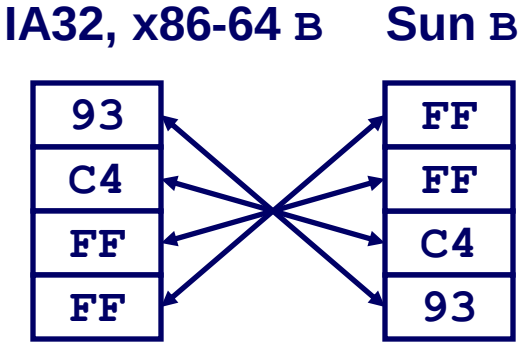
```
int a = 15213;
0x11ffffcb8   0x6d
0x11ffffcb9   0x3b
0x11ffffcba   0x00
0x11ffffcbb   0x00
```

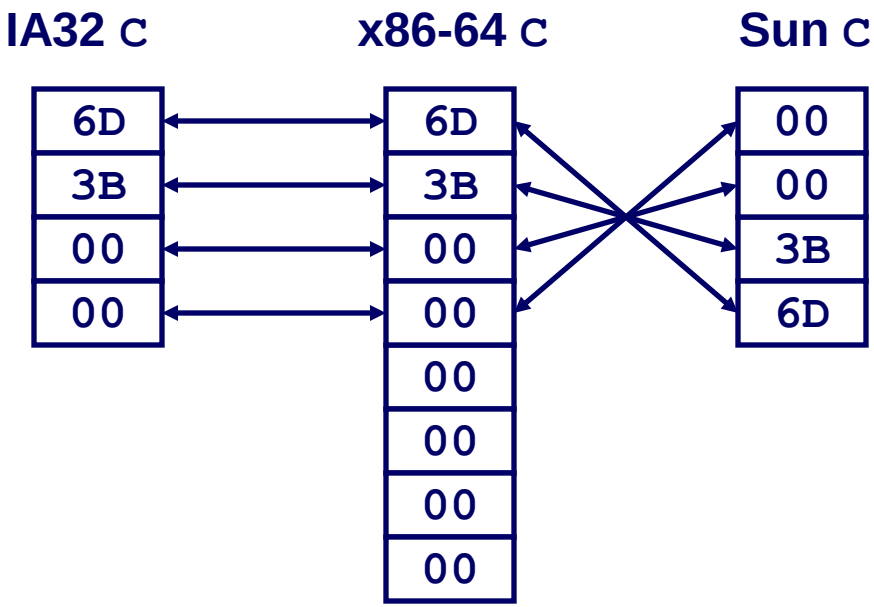
Representing Integers

```
int A = 15213;
int B = -15213;
long int C = 15213;
```

Decimal:	15213
Binary:	0011 1011 0110 1101
Hex:	3 B 6 D



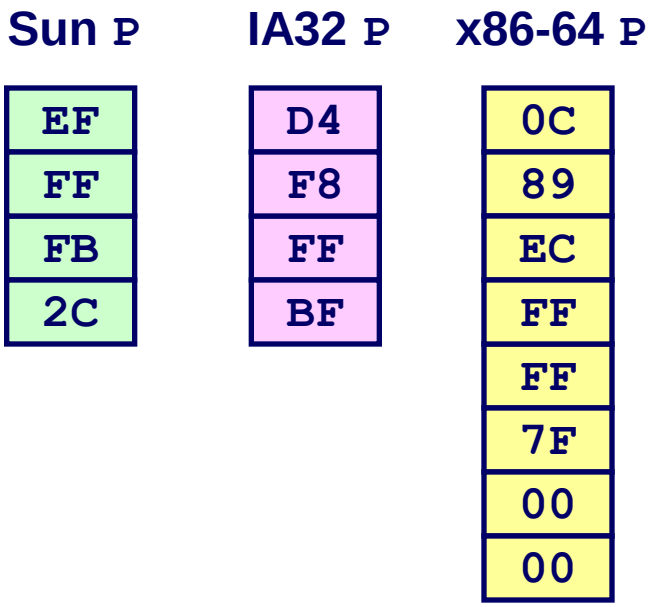




Two's complement representation (Covered later)

Representing Pointers

```
int B = -15213;
int *P = &B;
```



Different compilers & machines assign different locations to objects

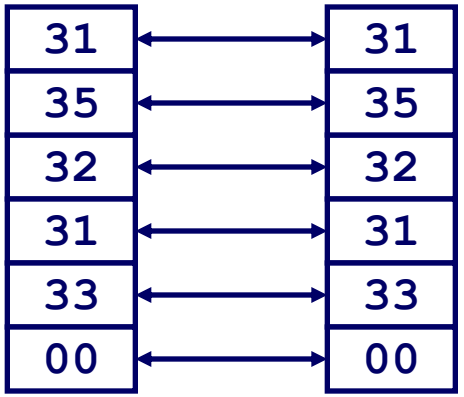
Representing Strings

```
char S[6] = "15213";
```

Strings in C

- Represented by array of characters
- Each character encoded in ASCII format
  - Standard 7-bit encoding of character set
  - Character "0" has code `0x30`
    - » Digit *i* has code `0x30`+*i*
- String should be null-terminated
  - Final character = 0

Linux/Alpha `S` Sun `S`



Compatibility

- Byte ordering not an issue

Boolean Algebra

Developed by George Boole in 19th Century

- Algebraic representation of logic
  - Encode "True" as 1 and "False" as 0

And

- A&B = 1 when both A=1 and B=1

&	0	1
0	0	0
1	0	1

Or

- A|B = 1 when either A=1 or B=1

|	0	1
0	0	1
1	1	1

Not

- ~A = 1 when A=0

~	
0	1
1	0

Exclusive-Or (Xor)

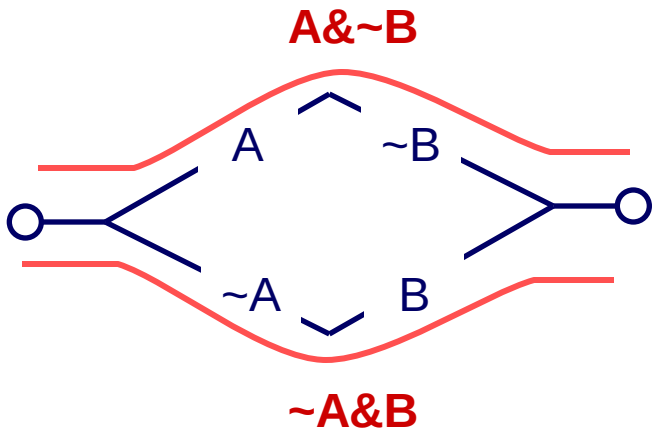
- A^B = 1 when either A=1 or B=1, but not both

^	0	1
0	0	1
1	1	0

Application of Boolean Algebra

Applied to Digital Systems by Claude Shannon

- 1937 MIT Master's Thesis
- Reason about networks of relay switches
  - Encode closed switch as 1, open switch as 0



Connection when

A&~B | ~A&B

= A^B

General Boolean Algebras

Operate on Bit Vectors

- Operations applied bitwise

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
  --------    | --------    ^ --------    ~ --------
  01000001      01111101      00111100      10101010
```

All of the Properties of Boolean Algebra Apply

Representing & Manipulating Sets

Representation

- **Width *w* bit vector represents subsets of** {0, …, *w*–1}
- $a_j = 1$ **if** $j \in A$

```
01101001          { 0, 3, 5, 6 }
76543210

01010101          { 0, 2, 4, 6 }
76543210
```

Operations

&	**Intersection**	`01000001`	{ 0, 6 }
|	**Union**	`01111101`	{ 0, 2, 3, 4, 5, 6 }
^	**Symmetric difference**	`00111100`	{ 2, 3, 4, 5 }
~	**Complement**	`10101010`	{ 1, 3, 5, 7 }

Bit-Level Operations in C

Operations &, |, ~, ^ Available in C

- Apply to any "integral" data type
  - `long, int, short, char, unsigned`
- View arguments as bit vectors
- Arguments applied bit-wise

Examples (Char data type)

- `~0x41 --> 0xBE`
  $\sim 01000001_2 \dashrightarrow 10111110_2$
- `~0x00 --> 0xFF`
  $\sim 00000000_2 \dashrightarrow 11111111_2$
- `0x69 & 0x55 --> 0x41`
  $01101001_2 \;\&\; 01010101_2 \dashrightarrow 01000001_2$
- `0x69 | 0x55 --> 0x7D`
  $01101001_2 \;|\; 01010101_2 \dashrightarrow 01111101_2$

Contrast: Logic Operations in C

Contrast to Logical Operators

- `&&, ||, !`
  - View 0 as "False"
  - Anything nonzero as "True"
  - Always return 0 or 1
  - Early termination

Examples (char data type)

- `!0x41  -->  0x00`
- `!0x00  -->  0x01`
- `!!0x41 -->  0x01`

- `0x69 && 0x55  -->  0x01`
- `0x69 || 0x55  -->  0x01`
- `p && *p` (avoids null pointer access)

Shift Operations

Left Shift: `x << y`

- Shift bit-vector `x` left `y` positions
  - » Throw away extra bits on left
  - Fill with 0's on right

Argument `x`	01100010
<< 3	00010*000*
Log. >> 2	*00*011000
Arith. >> 2	*00*011000

Right Shift: `x >> y`

- Shift bit-vector `x` right `y` positions
  - Throw away extra bits on right
- Logical shift
  - Fill with 0's on left
- Arithmetic shift
  - Replicate most significant bit on right

Argument `x`	10100010
<< 3	00010*000*
Log. >> 2	*00*101000
Arith. >> 2	*11*101000

Undefined Behavior

- Shift amount < 0 or ≥ word size

Integer C Puzzles

- Assume 32-bit word size, two's complement integers
- For each of the following C expressions, either:
  - Argue that is true for all argument values
  - Give example where not true

Initialization

```
int x = foo();
int y = bar();
unsigned ux = x;
unsigned uy = y;
```

- `x < 0` ⇒ `((x*2) < 0)`
- `ux >= 0`
- `x & 7 == 7` ⇒ `(x<<30) < 0`
- `ux > -1`
- `x > y` ⇒ `-x < -y`
- `x * x >= 0`
- `x > 0 && y > 0` ⇒ `x + y > 0`
- `x >= 0` ⇒ `-x <= 0`
- `x <= 0` ⇒ `-x >= 0`
- `(x|-x)>>31 == -1`
- `ux >> 3 == ux/8`
- `x >> 3 == x/8`
- `x & (x-1) != 0`

Encoding Integers

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

```
short int x =  15213;
short int y = -15213;
```

- C `short` 2 bytes long

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
y	-15213	C4 93	11000100 10010011

Sign Bit

- For 2's complement, most significant bit indicates sign
  - 0 for nonnegative
  - 1 for negative

Encoding Example (Cont.)

```
x =      15213: 00111011 01101101
y =     -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**